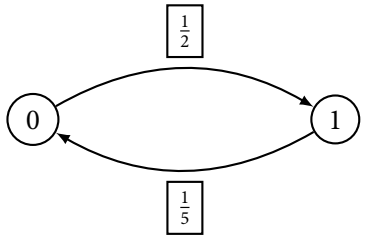
Solutions to Problem 1.

a.
- **State space.** $\mathcal{M} = \{0, 1\}$. State 0 represents being on the phone, state 1 represents answering email.
- **Transition rate diagram.** Time units are minutes.



- **Generator matrix.**

$$\mathbf{G} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{5} & -\frac{1}{5} \end{bmatrix}$$

b.

$$\left.\begin{aligned} -\frac{1}{2}\pi_0 + \frac{1}{5}\pi_1 &= 0 \\ \frac{1}{2}\pi_0 - \frac{1}{5}\pi_1 &= 0 \\ \pi_0 + \pi_1 &= 1 \end{aligned}\right\} \quad \Rightarrow \quad \pi_0 = \frac{2}{7}, \pi_1 = \frac{5}{7}$$

In the long-run, the customer service representative spends 2/7 of their time on the phone, and 5/7 of their time answering email.

Solutions to Problem 2.

a.
- **State space.** $\mathcal{M} = \{0, 1, 2\}$. States represent number of vans in use.
- **Generator matrix.** Time units are days.

$$\mathbf{G} = \begin{bmatrix} -\frac{8}{7} & \frac{8}{7} & 0 & 0 \\ \frac{1}{2} & -\frac{23}{14} & \frac{8}{7} & 0 \\ 0 & 1 & -\frac{15}{7} & \frac{8}{7} \\ 0 & 0 & \frac{3}{2} & -\frac{3}{2} \end{bmatrix}$$

b.

$$\left.\begin{aligned} -\frac{8}{7}\pi_0 + \frac{1}{2}\pi_1 &= 0 \\ \frac{8}{7}\pi_0 - \frac{23}{14}\pi_1 + \pi_2 &= 0 \\ \frac{8}{7}\pi_1 - \frac{15}{7}\pi_2 + \frac{3}{2}\pi_3 &= 0 \\ \frac{8}{7}\pi_2 - \frac{3}{2}\pi_3 &= 0 \\ \pi_0 + \pi_1 + \pi_2 + \pi_3 &= 1 \end{aligned}\right\} \quad \Rightarrow \quad \pi_0 \approx 0.127, \pi_1 \approx 0.290, \pi_2 \approx 0.331, \pi_3 \approx 0.252$$

Requests are denied when there are 3 vans in use, which occurs π_3 of the time. Since there are 8/7 requests per day, requests are denied at a rate of $(8/7)\pi_3 \approx 0.288$ requests per day.

Solutions to Problem 3.

a.
- **State space.** $\mathcal{M} = \{0, 1, 2\}$. States represent the number of failed machines.
- **Generator matrix.** Time units are hours. Note that the repair rate is $1/24 \approx 0.04$ hours.

$$\mathbf{G} = \begin{bmatrix} -0.02 & 0.02 & 0 \\ 0.04 & -0.06 & 0.02 \\ 0 & 0.04 & -0.04 \end{bmatrix}$$

b.
$$\left.\begin{aligned} -0.02\pi_0 + 0.04\pi_1 &= 0 \\ 0.02\pi_0 - 0.06\pi_1 + 0.04\pi_2 &= 0 \\ 0.02\pi_1 - 0.04\pi_2 &= 0 \\ \pi_0 + \pi_1 + \pi_2 &= 1 \end{aligned}\right\} \quad \Rightarrow \quad \pi_0 \approx 0.57, \pi_1 \approx 0.29, \pi_2 \approx 0.14$$

Therefore, the long-run fraction of time that both testing machiens are not working is $\pi_2 \approx 0.14$.

c. The long-run fraction of time that at least one testing machine is not working is $\pi_1 + \pi_2 \approx 0.43$.